Long-range Response in AC Electricity Grids

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Abstract

Local changes in the topology of electricity grids can cause overloads far away from the disturbance [1], making the prediction of the robustness against changes in the topology – for example caused by power outages or grid extensions – a challenging task. The impact of single-line additions on the long-range response of DC electricity grids has recently been studied [2]. By solving the real part of the static AC load flow equations, we conduct a similar investigation for AC grids. In a regular 2D grid graph with cyclic boundary conditions, we find a power law decay for the change of power flow as a function of distance to the disturbance over a wide range of distances. The power exponent increases and saturates for large system sizes. By applying the same analysis to the German transmission grid topology, we show that also in real-world topologies a long-ranged response can be found.

Power flow equations of an inductive grid

→ Assume a purely *inductive* grid and sinusoidal voltages.

⇒ Constant nodal voltage magnitudes $|V_i| \equiv V$ [3].

⇒ Reactive power Q_i always balanced, so it is sufficient to solve the *active power flow equations*:

$$P_i = \sum_j K_{ij} \sin(\theta_i - \theta_j) \quad . \tag{1}$$

⇒ System of N nonlinear equations, solvable by a standard *root-finding algorithm*in order to find the *phase distribution* θ_i.

→ Definition of the *power capacity* of edge (i,j) [3]:

$$K_{ij} = \frac{V^2}{\omega L_{ij}} \quad . \tag{2}$$

→ Consider voltage *phase angles* $\varphi_i(\omega,t) = \omega t + \theta_i(t)$, with the *grid frequency* $\omega = 2\pi \cdot 50\,\text{Hz}$ [3].

→ Compute the *transmitted power*

$$F_{ij} = K_{ij} \sin(\theta_i - \theta_j) \tag{3}$$

for each edge (i,j) of the graph.

2D grid model

→ Cyclic square 2D grid graph, size $N = L^2$.

→ Binary distribution for nodal *net generated power* $P_i \in \{-P, +P\}$. Condition $\sum_i P_i = 0$ must be fulfilled, so the linear system size L must be an even number.

→ Consider constant *power capacities* $K_{ij} = K$.

→ Add another "diagonal" transmission line somewhere in the grid and observe the change of power flow $\Delta F_{ij} = F_{ij}^{\text{after}} - F_{ij}^{\text{before}}$ for each transmission line (i,j).

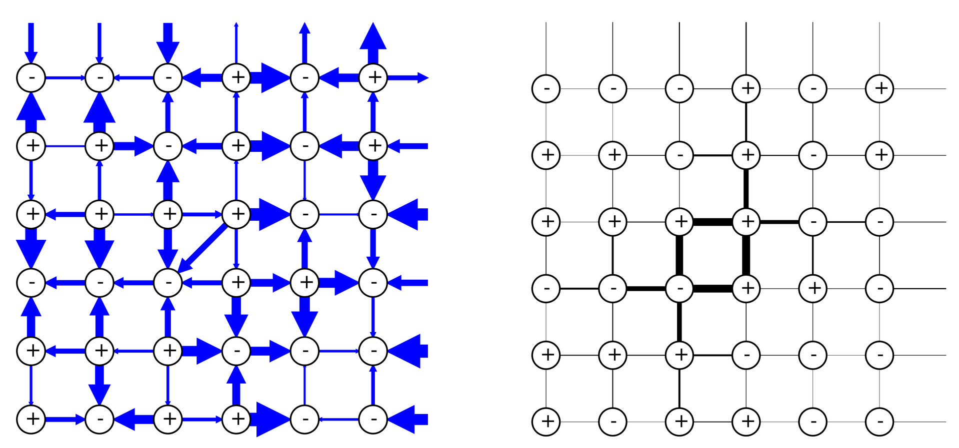

Figure 1: *(a) Power flow after the addition of another transmission line (diagonal at the center). (b) Change of transmitted power after adding the line. The width of each line is proportional to the absolute change of transmitted power, $|\Delta F_{ij}|$ [4].*

→ To analyze response properties, consider edge distance r_{ij} of the edge (i,j) to the added edge.

→ Average $|\Delta F_{ij}|$ over all edges (i,j) with the same distance r to the disturbance, and over $R = 1000$ realizations of disorder (random placement of generators and consumers). Realizations for which no stable solution can be found are skipped.

Long-range response in a 2D grid

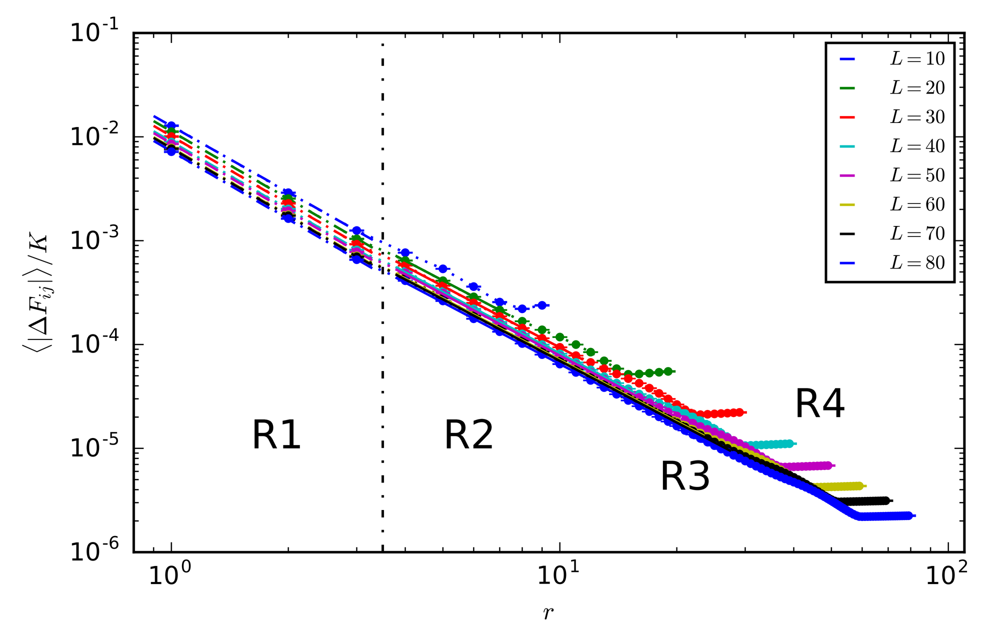

Figure 2: *Double-logarithmic plot of $\langle|\Delta F_{ij}|\rangle(r)$ for different linear system sizes L. For the regimes R1 ($r \le 3$) and R2 ($4 \le r < 2L/5$), the data has been fitted to a power law (4) [4].*

→ The distance regimes R1 and R2 are dominated by a *power law* behavior of $\langle|\Delta F_{ij}|\rangle(r)$ (see Fig. 2). We fit the data to the fit model

$$f(r) = a_k r^{-b_k} \quad , \tag{4}$$

where $k \in \{1,2\}$ (R1, R2). Fig. 4 shows fit results for R2 and their dependence on linear system size L and power ratio P/K.

→ In distance regime R3, the data is almost following an exponential law (not demonstrated here) [4].

→ In R4, $\langle|\Delta F_{ij}|\rangle$ saturates for $r \to L$ (or even slightly increases again).

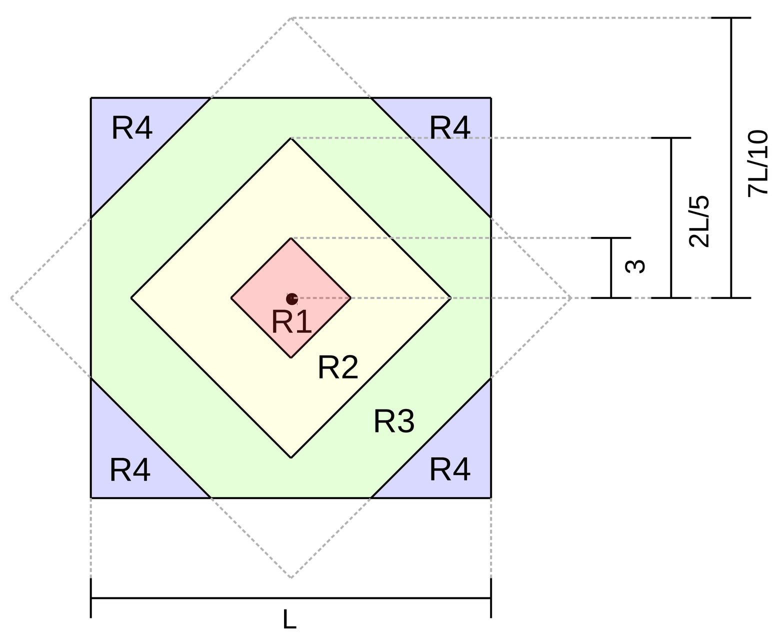

Figure 3: *Sketch of the distance regimes R1, R2, R3 and R4 in a cyclic $L \times L$ 2D grid, each with a different long-range behavior. Black dot: Location of the topological perturbation [4].*

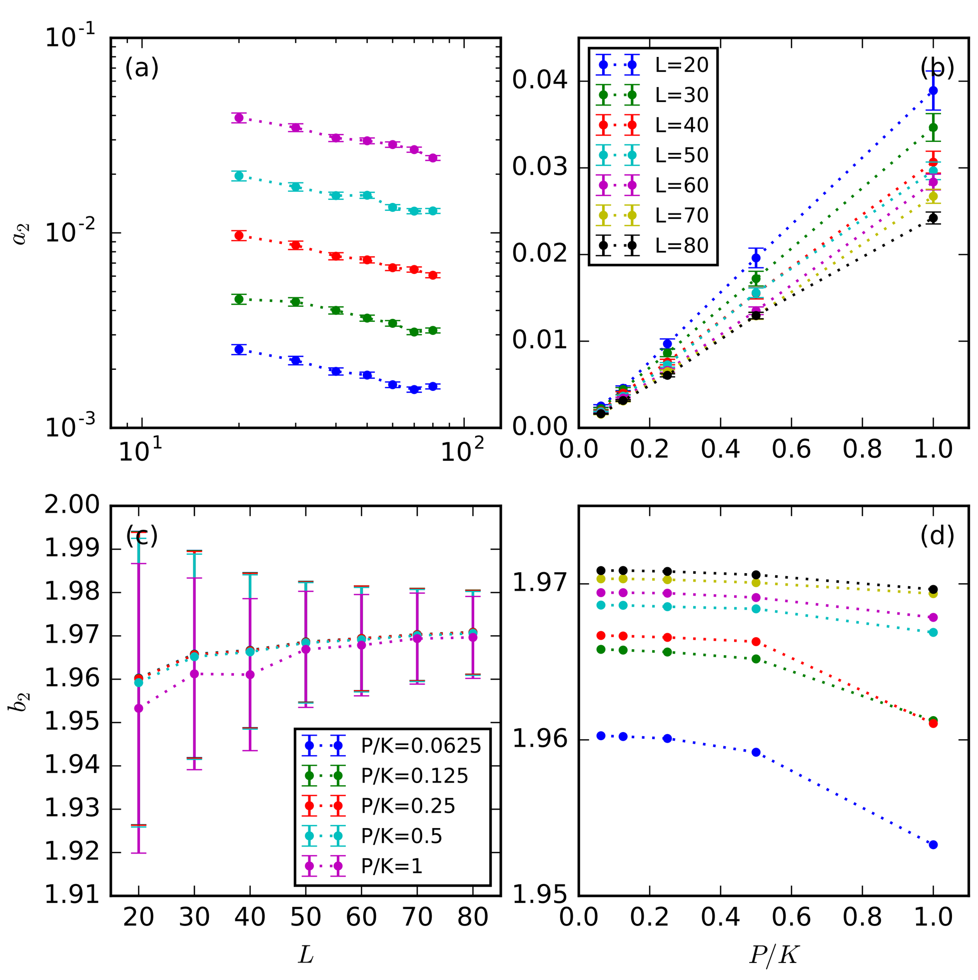

Figure 4: *Dependence of the fit parameters a_2 and b_2 (regime R2) on the linear system size L and the ratio P/K. For better readability, the error bars are not shown in the bottom right plot [4].*

References

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Long-range response in the German transmission grid

→ Load flow perturbations cannot pass 1-cut nodes (no rerouting possible).

⇒ Consider largest 2-connected component for this study.

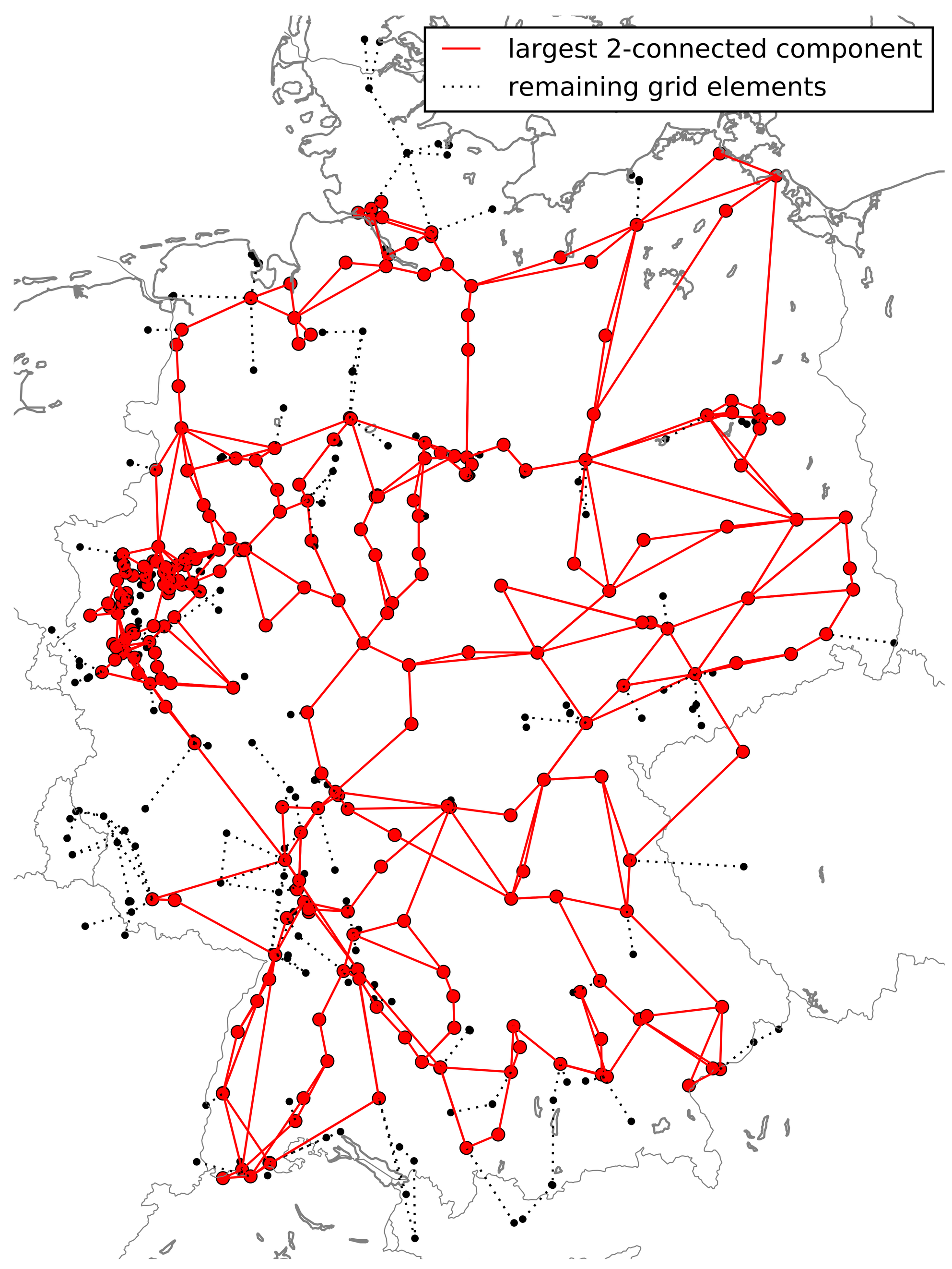

Figure 5: *Model for the German transmission grid (220 kV and 380 kV), based on SciGRID data [5]. The largest 2-connected component is marked in red [4].*

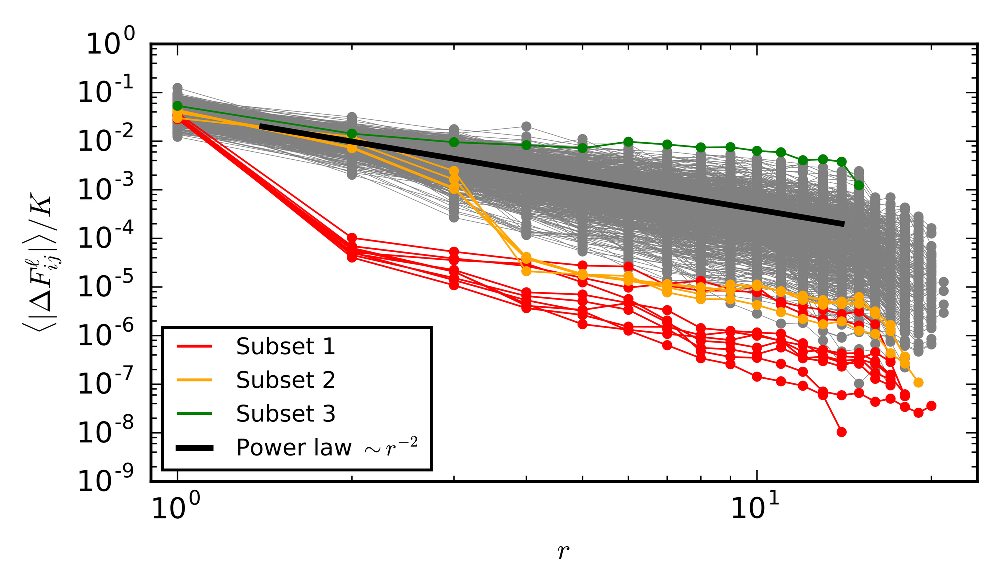

Figure 6: *Double-logarithmic plot of $\langle|\Delta F_{ij}^\ell|\rangle(r)$ for 880 single lines ℓ (gray) added to the largest 2-connected component of the German transmission grid model with $P/K = 0.25$, $w = 1$ and $R = 100$ [5]. For comparison, the thick black line illustrates a power law $\sim r^{-2}$. Some curves are highlighted with color, belonging to the different subsets of added edges in Fig. 7 [4].*

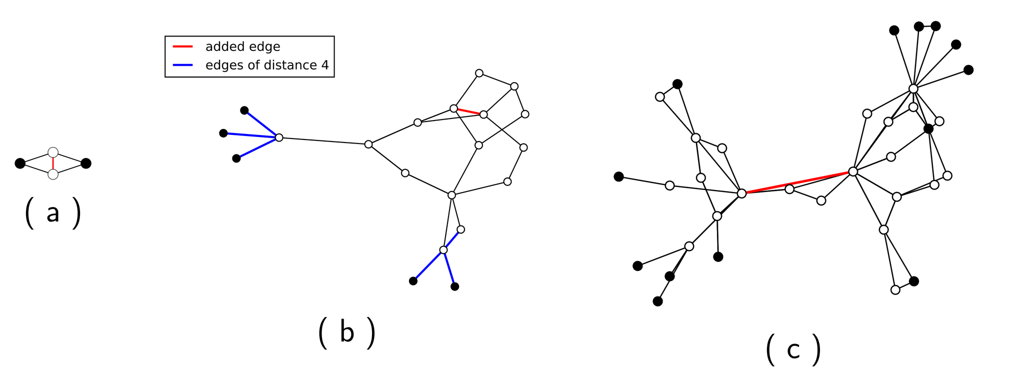

Figure 7: *Typical local topologies in the vicinity of the added edge (marked in red) leading to different responses: (a) Sudden drop of $\langle|\Delta F_{ij}|\rangle(r)$ at $r = 2$ (red curves in Fig. 6). (b) Sudden drop of $\langle|\Delta F_{ij}|\rangle(r)$ at $r = 4$ (orange curves in Fig. 6). (c) Profoundly weak decay of $\langle|\Delta F_{ij}|\rangle(r)$ (green curve in Fig. 6). Filled circles: Connecting nodes to the rest of the grid [4].*

Conclusions

→ We have shown numerically that local grid modifications cause a long-range response in AC electricity grids (power law).

→ The power exponent in the medium distance regime (R2) is saturating at a value of $b_2 \approx 1.971$ ($L = 80$).

→ German power grid: Decay rate depends strongly on the grid structure at different distances on each path, thus no pure power law.

Possible future work

→ Follow each path separately through irregular grids, find relation of decay rate to geometric measures along the path.

→ Study time-dependent spreading of local phase perturbations, following a recently published approach [3]. Clarify the role of *Anderson localization* in AC transmission grids.

→ Application to other random topologies.

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