Collective Nonlinear Dynamics of Electricity Networks (CoNDyNet)

STROMNETZE Forschungsinitiative der Bundesregierung

Determination of resonance frequencies of LC networks with binary link disorder

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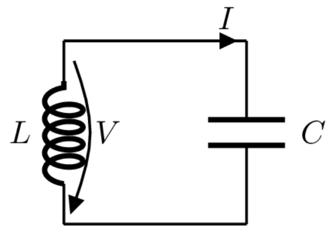
Table of contents

Motivation

A *LC circuit* has a *resonance frequency* of $\omega_0 = \frac{1}{\sqrt{LC}}$



Frequency ω of AC current approaching the resonance frequency:

$$\lim_{\omega \to \omega_0} Z(\omega) = \infty$$

$$Z = R + iX$$

$$\frac{1}{X} = \frac{1}{i\omega L} + i\omega C$$

$\Rightarrow$ **No power transmission possible!**

Normal operation: ω should stay *far below* the smallest resonance.

Arbitrary LC network

- Coupled LC oscillators
- Set of resonance frequencies ω_n

Flow equations I

Combine *Ohm's law*

$$V_{ij} = Z_{ij} I_{ij} \quad \text{or} \quad I_{ij} = Y_{ij} V_{ij}$$

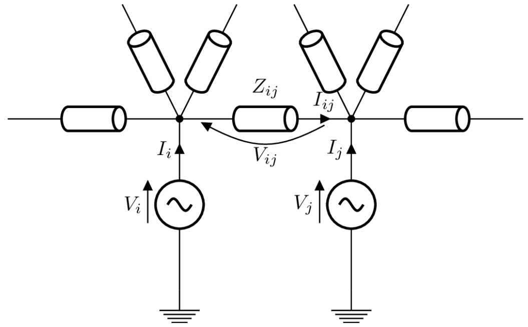
with *Kirchhoff's laws* for each node and mesh

$$I_i = \sum_j I_{ij} \qquad V_{ij} = V_i - V_j$$

to derive the *current flow equations*

$$I_i = \sum_j Y_{ij}(V_i - V_j) \quad .$$

Name	**Symbol**
Impedance matrix	$\mathbf{Z}$
Admittance matrix	$\mathbf{Y}$



An arbitrary one-phasic AC grid with line impedances Z_{ij}.

Generator: $I_i > 0$
Consumer: $I_i < 0$

$$\mathbf{Y} = \mathbf{Z}^{-1}$$

R. Huang, G. Korniss, S. Nayak, 2009 (http://dx.doi.org/10.1103/PhysRevE.80.045101)

Flow Equations II

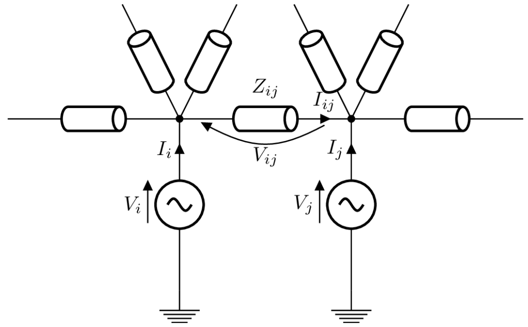
$$I_i = \sum_j Y_{ij}(V_i - V_j)$$

To get a standard matrix-vector multiplication, reformulate to

$$I_i = \sum_j L_{ij} V_j \quad \text{or} \quad \mathbf{I} = \mathbf{L}\mathbf{V}$$

with

$$L_{ij} = \delta_{ij} \sum_{k \neq i} Y_{ik} - (1 - \delta_{ij}) Y_{ij}$$



An arbitrary one-phasic AC grid.

Generator: $I_i > 0$
Consumer: $I_i < 0$

Note:

- $\mathbf{L}$ is defined in analogy to the *topological network Laplacian* $\mathbf{G}$.
- $\mathbf{L}$ is commonly referred to as *admittance matrix* as well.

R. Huang, G. Korniss, S. Nayak, 2009 (http://dx.doi.org/10.1103/PhysRevE.80.045101)

Basic Graph Theory I

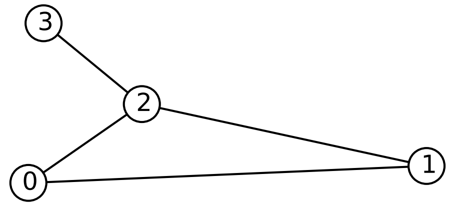
A *graph* is given by

- a set of *nodes* i
- a set of *edges* (i, j)

Properties

Property	Explanation
Order N	Number of nodes
Size	Number of edges

Example



$$N = 4$$

Basic Graph Theory II

Node Properties

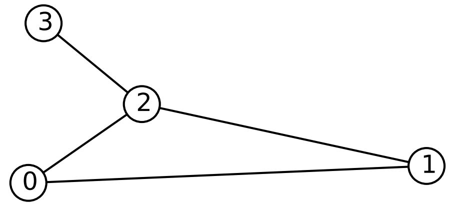
Property	Explanation
Degree	Number of incident edges

Matrices

Name	Explanation
Degree matrix $\mathbf{D}$	Diagonal matrix containing all node degrees
Adjacency matrix $\mathbf{E}$	Nonzero only if nodes i and j are *adjacent*
Laplacian matrix $\mathbf{G}$	$\mathbf{G} = \mathbf{D} - \mathbf{E}$

$$G_{ij} = \delta_{ij} \sum_{k \neq i} E_{ik} - (1 - \delta_{ij}) E_{ij}$$

Example



$$D = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$E = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$G = \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

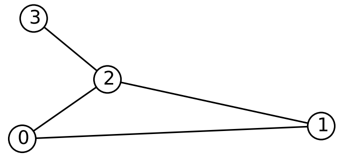
Basic Graph Theory III

Laplacian spectrum

Certain eigenvalues (EVs) of the Laplacian matrix $\mathbf{G}$ have special properties:

- All EVs are non-negative (as $\mathbf{G}$ is *positive semi-definite*).
- At least one eigenvalue is 0.
- Number of eigenvalues equal to 0: Number of *connected subgraphs*.
- Second-smallest EV: *Algebraic connectivity*.

Example

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Laplacian spectrum:
[0.  1.  3.  4.]
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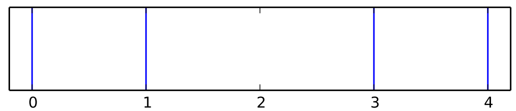
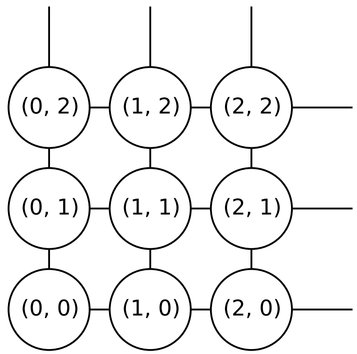


$$G = \begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & -1 & 0 \\ -1 & -1 & 3 & -1 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

The regular 2D grid graph



$N = 9$, periodic boundary conditions

The Small World Model

Closed ring with N nodes, plus S randomly chosen *shortcuts*.

Shortcut density $\frac{p}{2} = \frac{S}{N}$

Number of shortcuts $S = \frac{Np}{2}$

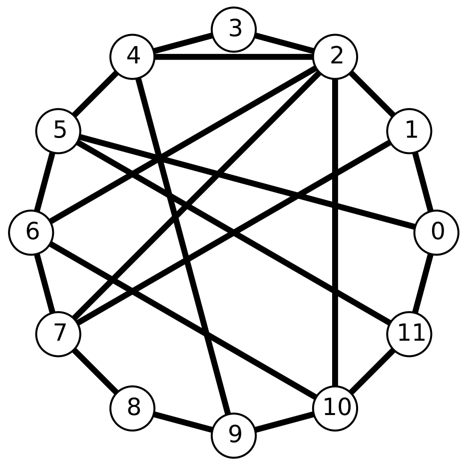
Two limiting cases:

Limiting case	Resulting graph
$p_{\min} = 0$	*Closed ring*
$p_{\max} = N - 3$	*Complete graph*

Maximum number of shortcuts:

$$S_{\max} = \frac{N(N-3)}{2}$$

Example



$N = 12 \qquad p = 1.5$

J. Travers, S. Milgram, 1969 (http://www.jstor.org/stable/2786545); M. Newman, D. Watts, 1999 (http://dx.doi.org/10.1016/S0375-9601%2899%2900757-4); R. Huang, G. Korniss, S. Nayak, 2009 (http://dx.doi.org/10.1103/PhysRevE.80.045101)

Adding Edge Attributes

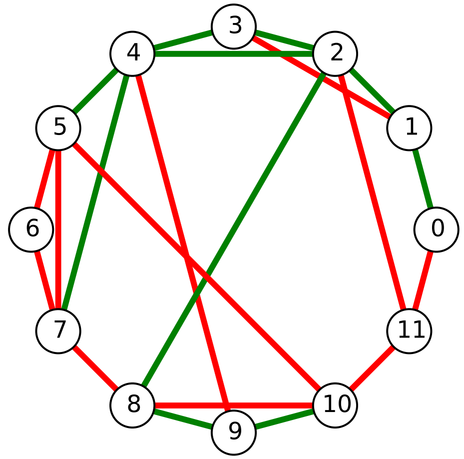
In order to describe *LC networks*, we attribute an *impedance* Z_{ij}
to each edge.

Here, we consider *random* impedances, using a *binary distribution*:

Edge type	$\boldsymbol{Z_{ij}}$	**Chance**
Capacitance	$(i\omega C)^{-1}$	q
Inductance	$i\omega L$	$1-q$

R. Huang, G. Korniss, S. Nayak, 2009 (http://dx.doi.org/10.1103/PhysRevE.80.045101)

Example



$$N = 12$$
$$p = 1.5 \qquad q = 0.5$$

Edge type	$\boldsymbol{Z_{ij}}$	**Chance**	$\boldsymbol{Y_{ij}}$	$\boldsymbol{h_{ij}}$
Capacitance	$(i\omega C)^{-1}$	q	$y_1 = i\omega C$	-1
Inductance	$i\omega L$	$1-q$	$y_2 = (i\omega L)^{-1}$	1

Introduce matrix $\mathbf{h}$:

$$h_{ij} = \begin{cases} -1 & , \quad \text{edge } (i,j) \text{ carries capacitance C} \\ 1 & , \quad \text{edge } (i,j) \text{ carries inductance L} \\ 0 & , \quad \text{no edge between i and j.} \end{cases}$$

$\Rightarrow$ Similar to $\mathbf{E}$, but also -1 allowed.

R. Huang, G. Korniss, S. Nayak, 2009 (http://dx.doi.org/10.1103/PhysRevE.80.045101)

Resonance frequencies I

Remember:

Flow equations:

$$I_i = \sum_j L_{ij} V_j \quad \text{or}$$

$$\mathbf{I} = \mathbf{L}\mathbf{V}$$

with "Laplacian matrix" $\mathbf{L}$.

- Consider *resonance case*, $I_i = 0$
- The system can be seen as a *system of coupled LC oscillator circuits*
- The system has N resonance frequencies ω_n

$$\sum_j L_{ij}(\omega)\, V_j = 0 \qquad \text{or} \qquad \mathbf{L}\mathbf{V} = \mathbf{0}$$

R. Huang, G. Korniss, S. Nayak, 2009 (http://dx.doi.org/10.1103/PhysRevE.80.045101)

Resonance frequencies II

Remember:

$$y_1 = i\omega C$$
$$y_2 = (i\omega L)^{-1}$$

$$h_{ij} = \begin{cases} -1 & , \quad \mathrm{Y_{ij} = y_1} \\ 1 & , \quad \mathrm{Y_{ij} = y_2} \\ 0 & , \quad \mathrm{Y_{ij} = 0} \end{cases}$$

Define $\mathbf{H}$,

$$H_{ij} = \delta_{ij} \sum_{k \neq i} h_{ik} - (1 - \delta_{ij}) h_{ij} \quad ,$$

and

$$\lambda = \frac{y_1 + y_2}{y_1 - y_2} \quad ,$$

so that the *flow equations for the resonance case* can be rewritten as

$$\mathbf{LV} = \mathbf{0} \qquad \Rightarrow \qquad (\mathbf{H} - \lambda \mathbf{G})\mathbf{V} = \mathbf{0} \quad .$$

But this is not yet a regular eigenvalue problem...

R. Huang, G. Korniss, S. Nayak, 2009 (http://dx.doi.org/10.1103/PhysRevE.80.045101); Fyodorov 1999 (http://dx.doi.org/10.1088/0305-4470/32/42/314)

Resonance frequencies III

Remember:

$$\lambda = \frac{y_1 + y_2}{y_1 - y_2}$$

$$y_1 = i\omega C$$
$$y_2 = (i\omega L)^{-1}$$

Define

$$\tilde{\mathbf{H}} = \mathbf{G}^{-1/2}\,\mathbf{H}\,\mathbf{G}^{-1/2}$$

(real symmetric) and

$$\tilde{\mathbf{V}} = \mathbf{G}^{1/2}\mathbf{V} \quad .$$

Notes:

1. $\mathbf{G}$ is **real** and **positive semi-definite**, so its *matrix square root* $\mathbf{G}^{1/2}$ is uniquely defined.
2. $\mathbf{G}$ is **positive semi-definite**, i.e. it has at least one eigenvalue 0 and hence is always *singular*. So its *pseudo-inverse* $\mathbf{G}^{-1}$ has to be considered.

Fyodorov 1999 (http://dx.doi.org/10.1088/0305-4470/32/42/314)

Resonance frequencies IV

Remember:

$$\lambda = \frac{y_1 + y_2}{y_1 - y_2}$$

$$y_1 = i\omega C$$
$$y_2 = (i\omega L)^{-1}$$

Define

$$\tilde{\mathbf{H}} = \mathbf{G}^{-1/2}\,\mathbf{H}\,\mathbf{G}^{-1/2}$$

(real symmetric) and

$$\tilde{\mathbf{V}} = \mathbf{G}^{1/2}\mathbf{V} \quad .$$

Then, we can rewrite

$$(\mathbf{H} - \lambda\mathbf{G})\mathbf{V} = \mathbf{0} \qquad \Rightarrow \qquad \tilde{\mathbf{H}}\,\tilde{\mathbf{V}}_n = \lambda_n \tilde{\mathbf{V}}_n$$

So we are facing a *regular eigenvalue problem*, with a known relationship between the eivenvalues λ_n
and the resonance frequencies ω_n:

$$\omega_n = \frac{1}{\sqrt{LC}}\sqrt{\frac{1+\lambda_n}{1-\lambda_n}}$$

Fyodorov 1999 (http://dx.doi.org/10.1088/0305-4470/32/42/314)

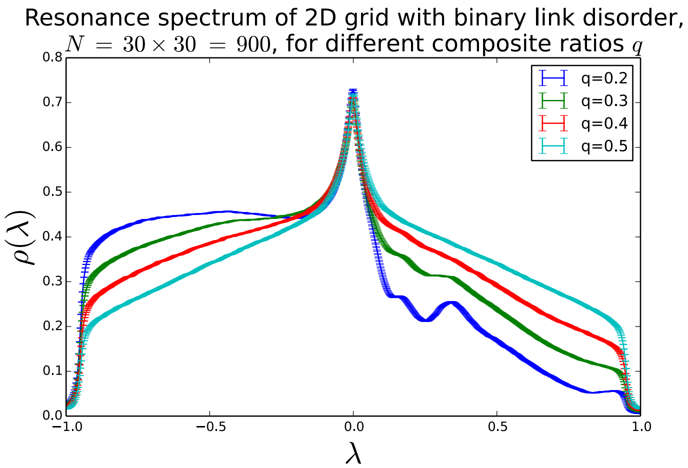
Resonance spectra I: 2D grid

Define the *density of resonances* (DOR):

$$\rho(\lambda) = \frac{1}{N} \sum_{n=1}^{N_{\mathrm{R}}} \delta(\lambda - \lambda_n)$$

N_{R} :
Number of "true" resonances
$(-1 < \lambda_n < 1)$

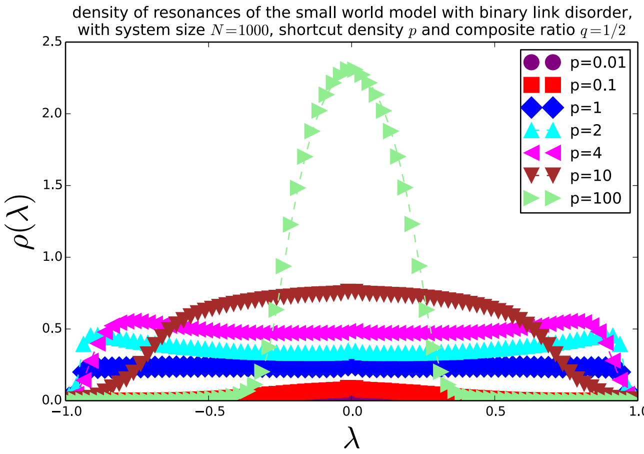
Obtain *ensemble average* (arithmetic mean) of the DOR over many disorder realizations (ADOR).

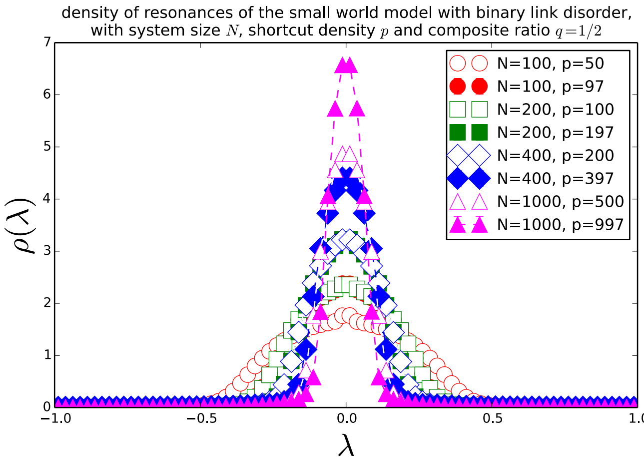


R. Huang, G. Korniss, S. Nayak, 2009 (http://dx.doi.org/10.1103/PhysRevE.80.045101)

Resonance spectra II: Small World Model

Large-p limit



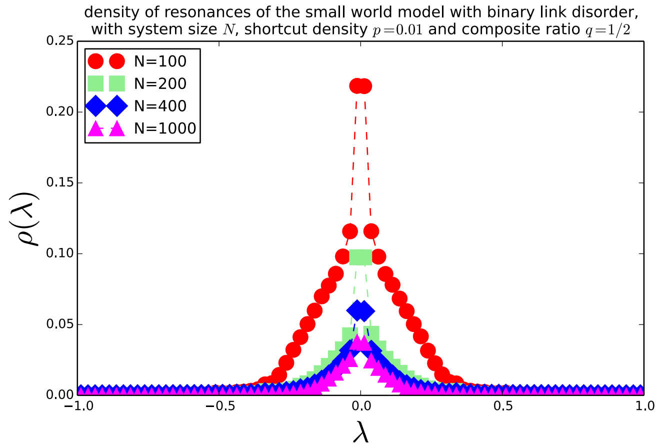


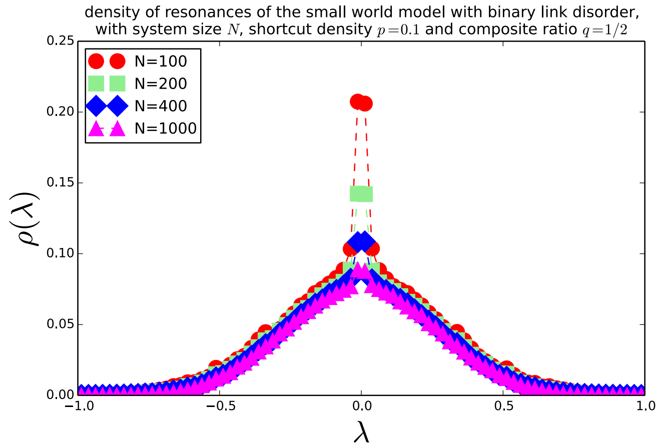
$\Rightarrow$ Confirming results by Huang et al. (http://dx.doi.org/10.1103/PhysRevE.80.045101)

R. Huang, G. Korniss, S. Nayak, 2009 (http://dx.doi.org/10.1103/PhysRevE.80.045101)

Resonance spectra III: Small World Model

Small-p limit



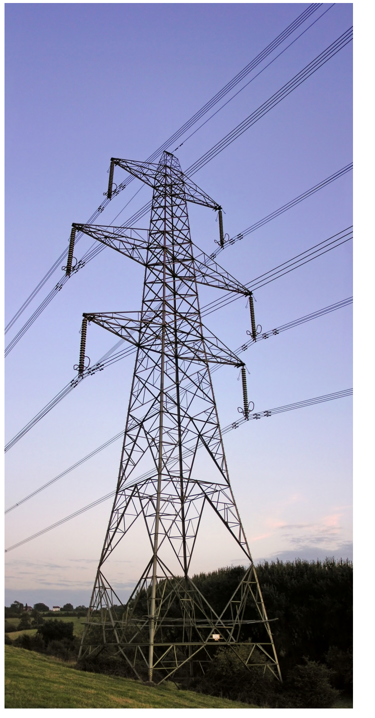


$\Rightarrow$ Largely confirming results by Huang et al. (http://dx.doi.org/10.1103/PhysRevE.80.045101)

$\Rightarrow$ Difference: Peak at $\lambda = 0$.

R. Huang, G. Korniss, S. Nayak, 2009 (http://dx.doi.org/10.1103/PhysRevE.80.045101)

Summary & Outlook



Summary

- Description of LC networks
  - simple graphs
  - binary distribution of edge impedances
- Calculation of *resonance frequencies* and the *density of resonances*

Outlook

- Different topologies (triangular grid, honeycomb grid, and **realistic network topologies**).
- Other impedance distributions (also **continuous distributions**), also including **ohmic resistances**.
- Beyond the resonance case (**current and power flow calculations**).

Thank you for your attention!

References

- J. Travers, S. Milgram, 1969 (http://www.jstor.org/stable/2786545)
- M. Newman, D. Watts, 1999 (http://dx.doi.org/10.1016/S0375-9601%2899%2900757-4)
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